Sensitivity of Doppler Radar with Self-Detecting Diode Oscillators

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Abstract—**The behavior of Doppler radar with self-detecting diode oscillators is described. Conditions for the frequency deviation to contain only the first harmonic of the Doppler shift frequency are given. The conversion gain and the signal-to-noise (S/N) ratio including both intrinsic LF and RF noise are calculated. An equivalent LF circuit is obtained. The shape of the I–V characteristic of the oscillating diode is shown to be important for the design of a Doppler module.**

I. INTRODUCTION

THE negative-resistance oscillator can be used in a Doppler radar as a self-detecting oscillator. A change in the reflected signal can be regarded as a varying load for the oscillator which induces a change in the bias current and voltage. The conversion gain from RF to LF can be larger than unity. The sensitivity is limited by the LF noise in the bias circuit and by RF noise. A noise figure of 25 dB, 30 kHz from the carrier, for a Doppler radar system with a Baritt diode has been measured [1].

Oscillator noise has been analyzed by several authors. Kurokawa [2] formulated a theory for AM and FM noise in oscillators. Thaler *et al.* [3] included noise from bias current oscillations. Vlaardingerbroek [4] discussed up-conversion and down-conversion of RF noise. Conversion gain for Doppler signal detection with negative resistance diode oscillators has been calculated by Nagano and Akaiwa [5]. Takayama [6] has derived a general theory for self-detection. A theory for frequency demodulation of Gunn and IMPATT diodes has been given by Bestwick *et al.* [7].

The purpose of this paper is to give a treatment of self-detection in diode oscillators when noise is taken into account. The time delay from oscillator to target is considered which has not been taken into account previously. An expression for the signal-to-noise ratio is presented. The influence of the equivalent LF circuit, the conversion gain, and different noise sources are explained. The formulas are expressed in physical quantities which helps the designer to choose a suitable diode and predict the properties of a Doppler module.

In Section II the frequency variation of the oscillator is discussed. A more detailed treatment is given in the Appendix. The signal-to-noise (S/N) ratio is derived in Section III where the reflected signal is considered to be equivalent to a time-dependent perturbation of the load. The expression for the S/N ratio is analyzed in Section IV and operating conditions for eliminating the influence of the LF noise source are determined. The case of a signal from a separate signal source is discussed in Section V.

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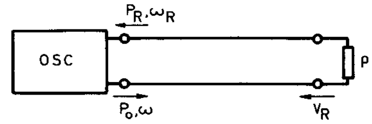

Fig. 1. Schematic diagram of a Doppler radar.

II. FREQUENCY VARIATION OF A SELF-DETECTING DIODE OSCILLATOR

Fig. 1 illustrates a self-detecting Doppler radar. It is shown in the Appendix that the frequency of the oscillator $\omega(t)$ is given by

$$\omega(t) = \omega_0 + \frac{B}{2} \sin(\omega_D t + \varphi_0) \tag{1}$$

if

$$B/2 \sin \omega_D T_D/2 \ll \omega_D \tag{2}$$

where ω_0 is the oscillator frequency with no reflected signal, B is the equivalent locking bandwidth, ω_D is the Doppler shift, $T_D/2$ is the time delay from oscillator to reflector, and φ_0 is a constant. Equation (2) is always fulfilled if half the locking bandwidth is much less than the Doppler shift. It is also satisfied if both half the locking bandwidth and the Doppler shift are much less than twice the inverted time delay. If (2) is not valid the frequency variation will contain higher harmonics of ω_D.

III. CONVERSION GAIN AND S/N RATIO

The equivalent circuit of the oscillator is shown in Fig. 2. We assume that the circuit consists of one part for high frequencies and one for low frequencies connected via the diode. The LF part is assumed to be resistive with R_0 representing the bias resistance. V_b is the bias voltage over the diode. $Z_D = R_D + jX_D$ is the RF diode impedance and $Z_L = R_L + jX_L$ is the load impedance. The RF current is assumed to be

$$i(t) = A(t) \cos \omega t.$$

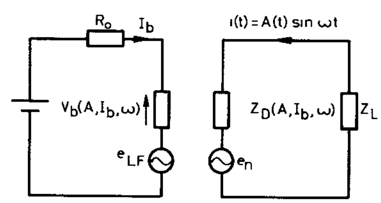

Fig. 2. Equivalent LF and RF circuits of a diode oscillator including noise sources.

The diode impedance Z_D and the bias voltage are functions of three parameters: the RF current amplitude A, the bias current I_b, and the oscillator frequency ω.

A. Derivation of LF Equivalent Circuit

Assuming that the equations in Section II are valid, the Doppler shifted signal can be represented by a load variation

$$\Delta Z_L(t) = r(\cos \omega_D t + j \sin \omega_D t)$$

where r is the magnitude of the impedance variations. The load variation ΔZ_L will cause a change in RF current by ΔA and in frequency by $\Delta\omega$. As there is coupling to LF, a bias current ΔI_b is obtained in the LF circuit. We assume a quasi-static oscillation, i.e., ΔZ_L varies slowly compared to the time constant of the RF circuit. This is always true for the usual applications of a Doppler radar. The following system of equations, where $Z = R + jX$, the total impedance, is the sum of Z_D and Z_L, can then be derived:

$$\frac{\partial Z}{\partial I_b} \Delta I_b(t) + \frac{\partial Z}{\partial \omega} \Delta\omega(t) + \frac{\partial Z}{\partial A} \Delta A(t) + r(\cos \omega_D t + j \sin \omega_D t) = 0 \quad (3)$$

$$\left(\frac{\partial V_b}{\partial I_b} + R_0\right) \Delta I_b(t) + \frac{\partial V_b}{\partial \omega} \Delta\omega(t) + \frac{\partial V_b}{\partial A} \Delta A(t) = 0. \quad (4)$$

Equations (3) and (4) state that the change in the total voltage is zero in the LF and RF circuits, respectively.

The derivatives of Z with respect to A, ω, and I_b are plotted in Fig. 3. The angle between $\partial Z/\partial A$ and $\partial Z/\partial\omega$ is defined by u and the angle between $\partial Z/\partial\omega$ and $\partial Z/\partial I_b$ by v. In Fig. 3 ΔZ_L is represented by a rotating vector with angular velocity ω_D. Equations (3) and (4) can then be used for calculating the rms value I_{be} of ΔI_b. The result is

$$I_{be}^2 = \frac{\alpha^2 r^2/2}{(R_0 + R_g)^2} \quad (5)$$

where

$$\alpha^2 = \frac{(\partial V_b/\partial A)^2}{|\partial Z/\partial A|^2 \sin^2 u} + \frac{(\partial V_b/\partial\omega)^2}{|\partial Z/\partial\omega|^2 \sin^2 u} - \frac{2(\partial V_b/\partial A)(\partial V_b/\partial\omega) \cos u}{|\partial Z/\partial A| |\partial Z/\partial\omega| \sin^2 u} \quad (6)$$

$$R_g = \frac{\partial V_b}{\partial I_b} + \frac{\partial V_b}{\partial A} \frac{|\partial Z/\partial I_b| \sin v}{|\partial Z/\partial A| \sin u} - \frac{\partial V_b}{\partial\omega} \frac{|\partial Z/\partial I_b| \sin (v + u)}{|\partial Z/\partial A| \sin u}. \quad (7)$$

The variable α describes how the rotating vector ΔZ_L causes a variation in the LF voltage V_b via variation in frequency and RF amplitude when $\Delta I_b = 0$. The first term depends on amplitude variations and the second on frequency variations. The third term describes the coupling between amplitude and frequency variations. This term is zero when they are out of phase, i.e., $u = \pi/2$. The equivalent LF resistance R_g given by (7) describes how V_b depends on the variations of I_b. R_g is the slope of the I–V characteristic of an oscillating diode. The first term is simply the I–V characteristic for constant frequency and amplitude. Variations in bias current will cause variations in amplitude and frequency which give LF voltage variations described by the second and third terms.

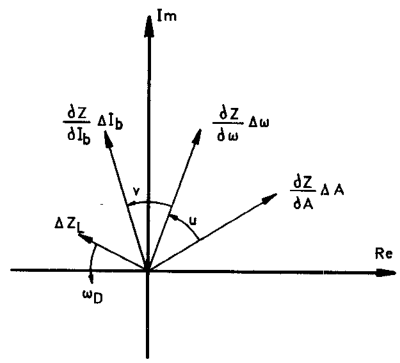

Fig. 3. The derivatives of $Z = Z_D + Z_L$ in a complex impedance plane. The rotating vector ΔZ_L with angular velocity ω_D represents the reflected signal.

For the special case where the diode voltage V_b is independent of frequency, i.e., $\partial V_b/\partial\omega = 0$, (6) and (7) can be written

$$\alpha^2 = \frac{(\partial V_b/\partial A)^2}{|\partial Z/\partial A|^2 \sin^2 u} \quad (8)$$

$$R_g = \frac{\partial V_b}{\partial I_b} + \frac{\partial V_b}{\partial A} \frac{|\partial Z/\partial I_b| \sin v}{|\partial Z/\partial A| \sin u}. \quad (9)$$

In Fig. 4 some experimental measurements of a Baritt diode are presented. The I–V characteristics are plotted for four load resistances, R_{L1}–R_{L4} where $R_{L1} > R_{L2} > R_{L3} > R_{L4}$. For $R = R_{L1}$ the diode does not oscillate which corresponds to the static curve. When R_L decreases, the RF amplitude increases, which lowers the dc voltage. R_g varies from zero to infinity and can even obtain negative values [8]. The dotted curve corresponds to maximum RF power. The dashed lines are mainly obtained from variation of the load resistance. The slope is simply determined by the bias resistance which is 20 Ω.

The noise sources are easily included. In the LF circuit the intrinsic noise voltage is $e_{LF}(t)$. In the RF circuit the noise source is represented by a noise impedance. The real and imaginary parts correspond to the noise voltage which is in phase and out of phase with the oscillator signal, respectively. It is given by

$$\begin{Bmatrix} R_n(t) \\ X_n(t) \end{Bmatrix} = \frac{2}{TA} \int_{t-T}^{t} e_n(\tau) \begin{Bmatrix} \cos \omega_0 \tau \\ \sin \omega_0 \tau \end{Bmatrix} d\tau \quad (10)$$

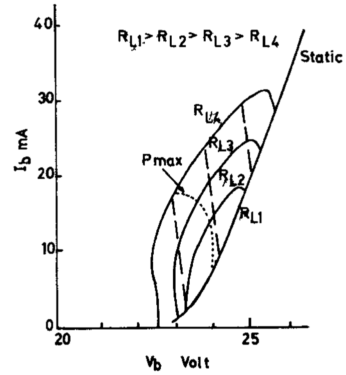

Fig. 4. Measured I–V characteristics for a Baritt diode with different load impedances. The dashed lines are obtained from load variation. The dotted curve corresponds to maximum output power. The frequency range is 7.2–7.4 GHz.

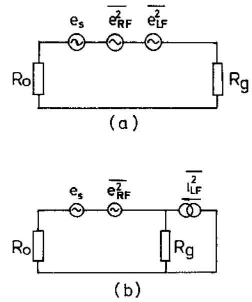

Fig. 5. Equivalent small signal LF circuits. (a) Intrinsic LF voltage noise source. (b) Intrinsic LF current noise source.

where $e_n(t)$ is the noise voltage and T is the period of the oscillation. The noise current $\overline{I_N^2}$ can be obtained from (3) and (4) by insertion of the noise impedance and assuming that $R_n(t)$ and $X_n(t)$ have equal mean square values. For a bandwidth of $d\Omega$ it is given by

$$\overline{I_N^2}(\Omega)\, d\Omega = \frac{[\overline{R_n^2}(\Omega)\alpha^2 + \overline{e_{LF}^2}(\Omega)]\, d\Omega}{(R_0 + R_g)^2}. \tag{11}$$

An equivalent LF circuit shown in Fig. 5(a) with signal generator and noise generators can then be defined where e_s represents the signal, $\overline{e_{RF}^2}(\Omega)$ the down-converted RF noise, and $\overline{e_{LF}^2}(\Omega)$ the intrinsic LF noise. From (5) it is obtained that

$$e_s^2 = \tfrac{1}{2} r^2 \alpha^2 \tag{12}$$

$$\overline{e_{RF}^2}(\Omega) = \overline{R_n^2}(\Omega)\alpha^2. \tag{13}$$

B. Conversion Gain

The conversion gain G is defined by

$$G = P_s/P_r \tag{14}$$

where P_s is the LF signal power in the bias resistance and P_r is the reflected RF power from the load. P_s is simply $I_{be}^2 R_o$ which can be written

$$P_s = \frac{e_s^2 R_0}{(R_0 + R_g)^2}. \tag{15}$$

The reflected power is obtained from $P_r = |\rho|^2 P_0$ where $|\rho| = (r/2R_L)$ and $P_o = \tfrac{1}{2} R_L A^2$. Equation (14) is then

$$G = \frac{4R_0}{(R_0 + R_g)^2} \times \frac{\alpha^2 R_L}{A^2}. \tag{16}$$

If R_g is positive, maximum power in the bias resistance is obtained for $R_0 = R_g$. If we neglect the frequency dependence of V_b, the gain can be written

$$G = \frac{(\partial V_b/\partial A)^2}{R_L R_g (A/R)^2\, |\partial Z/\partial A|^2 \sin^2 u}. \tag{17}$$

If we put $u = v = \pi/2$ and $\partial X/\partial A = \partial X/\partial I_b = 0$, it can be shown that (17) is identical to the expression for the conversion gain given by Takayama [6]. For maximum power (17) can be simplified further if $\partial X_D/\partial A = 0$. Then the following relation can be derived [9]:

$$\left(\frac{A}{R_D}\frac{\partial R_D}{\partial A}\right) = -2$$

which gives

$$G = \frac{(\partial V_b/\partial A)^2}{4 R_L R_g \sin^2 u}. \tag{18}$$

Equation (18) shows the importance of $\partial V_b/\partial A$ which expresses the rectifying capacity of the diode.

C. S/N Ratio

Using (12) and (13) the S/N ratio is given by

$$\mathrm{S/N} = \frac{\tfrac{1}{2} r^2 \alpha^2}{(\overline{R_n^2}(\Omega)\alpha^2 + \overline{e_{LF}^2}(\Omega))\, d\Omega}. \tag{19}$$

Thus the S/N ratio is the down-converted RF signal divided by the sum of the down-converted RF noise and the intrinsic LF noise. From (19) it is obvious that S/N is independent of the bias resistance R_0.

Expressed in the conversion gain, (19) can also be written

$$\mathrm{S/N} = \frac{G P_r}{(2GMkT_0 + [R_0/(R_0 + R_g)^2]\overline{e_{LF}^2}(\Omega))\, d\Omega} \tag{20}$$

where M is the noise measure. Typical values, 1 MHz from the carrier are for a Baritt oscillator 15 dB, for an avalanche diode oscillator 40 dB, and for a Gunn oscillator 25 dB. The factor 2 in the RF noise power enters because both sidebands are down-converted.

IV. Discussion of S/N Ratio

The expression for the S/N ratio in (20) depends on many parameters. We will concentrate our discussion on the conversion gain G, the equivalent LF resistance R_g, and the LF noise source. Different assumptions for the intrinsic LF noise source are made which give various optimum conditions.

In the first case we assume that the noise source can be represented by a voltage generator shown in Fig. 5(a) which only depends on the dc current. The S/N ratio is then

$$\mathrm{S/N} = P_r/2MkT_0 d\Omega, \qquad \text{for } R_g = \infty. \tag{21}$$

The influence of the intrinsic LF noise source is then eliminated. In order to get a large R_g it is necessary to let $|\partial Z/\partial A| \sin u$ go towards zero. The other derivatives in the expression for R_g in (9) are finite quantities. α^2 will then also become large which means that the conversion gain can be written

$$G = \frac{4R_0R_L}{A^2 \,|\, \partial Z/\partial I_b \,|^2 \sin^2 v}. \tag{22}$$

The diode then acts as an LF current generator.

The condition

$$\left| \frac{\partial Z}{\partial A} \right| \sin u = 0$$

means an unstable oscillator. However, if the interaction with the bias circuit is considered, it can be shown that there can exist stable oscillations even in this case, depending on the sign of $\partial V_b/\partial A \sin v$.

If the noise source is a current generator represented by i_{LF^2} [shown in Fig. 5(b)], (20) can be written

$$\mathrm{S/N} = \frac{GP_r}{(2GMkT_0 + [R_0R_g^2\overline{i_{\mathrm{LF}}^2}/(R_0 + R_g)^2])\, d\Omega}. \tag{23}$$

For $R_g = 0$ the influence of the intrinsic LF noise source is eliminated, which means that the current noise generator is short-circuited. The conversion gain G can still be finite so that the S/N ratio is of the same form as in (21).

As a conclusion, it is important to know the kind of LF noise source in order to optimize the S/N ratio. The extreme cases $R_g = 0$ or $R_g = \infty$ correspond to vertical and horizontal parts at the I–V characteristic, respectively.

It is not obvious that all diodes with negative resistance have such points. They exist for a Baritt diode which is shown in Fig. 4. Some Gunn diodes also have this property.

V. SIGNAL WITH CONSTANT FREQUENCY

We have, in previous sections, discussed the case where the received signal is a reflected part of the oscillator power. However, the treatment in Sections III and IV is also valid for mixing a signal from a separate signal source. The only restriction is that (2) must be fulfilled, i.e., the intermediate frequency is much larger than the corresponding locking bandwidth. The magnitude r of the impedance perturbation ΔZ_L must now also be redefined as

$$r = 2\frac{V_r}{A}$$

where V_r is the amplitude of the received signal normalized so that the received RF power is

$$P_r = \tfrac{1}{2}V_r^2/R_L.$$

VI. CONCLUSIONS

The behavior of the interaction of a Doppler signal and an oscillator has been demonstrated. The frequency deviation of the oscillator is half of the locking bandwidth corresponding to the reflected signal power. The modulation frequency is the Doppler frequency shift. The variation of the frequency contains only the first harmonic of the Doppler shift, if both the locking bandwidth and Doppler shift are much less than the inverted time delay to reflector. Another possibility for pure sinusoidal variation is that the locking bandwidth is much less than the Doppler shift.

The conversion gain and the LF equivalent diode resistance R_g have been calculated. Theory and experiments show that R_g can become negative. By choosing the bias resistance close to R_g, the conversion gain can be made arbitrarily high. This choice will not influence the S/N ratio which is independent of the bias resistance.

Expression for the S/N ratio has been given where the noise sources are the intrinsic LF and RF noise. Two cases are assumed for the LF noise source, either a voltage generator or a current generator which only depends on the bias current. The influence of the intrinsic LF noise source can be eliminated by a proper choice of R_g. For a voltage noise generator and a current noise generator this occurs for $R_g = \infty$ and $R_g = 0$, respectively.

APPENDIX

DERIVATION OF THE FREQUENCY VARIATIONS

In this Appendix the principles of self-detection are discussed. The effect on an oscillator of a moving load is investigated. Conditions for treating a self-detecting Doppler radar as an oscillator with a pure sinusoidal time-varying load are given.

The oscillator (see Fig. 1) is connected to a transmission line with a moving reflector with velocity v_R at the other end. The magnitude of the reflection factor ρ is small and the length of the transmission line corresponds to a time delay from oscillator to load of $T_D/2$. From (3) and (4) it is possible to derive an equation of the form

$$\dot{\varphi} = \omega_R - \omega_0 - \frac{B}{2}\sin\varphi \tag{24}$$

where

- φ phase difference between the received signal from the reflector and the oscillator signal;
- ω_R frequency of the received signal;
- ω_0 frequency of the oscillator for $|\rho| = 0$;
- B locking bandwidth.

Equation (24) was originally derived by Adler [10] and is valid for quasi-static oscillation. In his simplified treatment the locking bandwidth B is given by

$$B = \frac{2\omega_0}{Q_{\mathrm{ext}}}|\rho|$$

where Q_{ext} is the external Q value. When amplitude and bias current variations are taken into account, the expression for the locking bandwidth will be modified.

Since

$$\omega_R(t) = \omega(t - T_D) + \omega_D$$

where ω_D is the Doppler frequency shift, the phase difference φ can be expressed by

$$\varphi = \int_{t-T_D}^{t} \Delta\omega(\tau)\, d\tau + \omega_D t + \varphi_0 \qquad (25)$$

where φ_0 is a constant and $\Delta\omega = \omega - \omega_0$.

From (24) and (25) an implicit equation for the frequency of the oscillator can be derived:

$$\omega(t) = \omega_0 + \frac{B}{2} \sin \left[- \int_{t-T_D}^{t} \Delta\omega(\tau)\, d\tau + \omega_D t + \varphi_0 \right]. \qquad (26)$$

The solution to (26) will contain harmonic frequencies of ω_D. Taking into account only the first two harmonics and assuming that the integral in (26) is small, it can be shown by iteration that

$$\omega(t) = \omega_0 + \frac{B}{2} \left\{ \sin(\omega_D t + \varphi_0) - x \left[\sin 2\left(\omega_D t + \varphi_0 - \frac{\omega_D T_D}{4} \right) + \sin\left(\frac{\omega_D T_D}{2} \right) \right] \right\} \qquad (27)$$

where

$$x = \frac{B}{2\omega_D} \sin \omega_D T_D/2.$$

If $\omega_D T_D \ll 1$, this is the same expansion parameter as used by Takayama [6]. The second harmonic can be neglected if $x \ll 1$ which can be written

$$\frac{B}{2} \sin \frac{\omega_D T_D}{2} \ll \omega_D. \qquad (28)$$

Equation (28) is always fulfilled if

$$B/2 \ll \omega_D \qquad (29)$$

which means that half the locking bandwidth is much less than the Doppler shift. However, this is not true for Doppler signal detection in general. Equation (28) is also satisfied if

$$B/2 \ll 2/T_D \qquad (30)$$

and

$$\omega_D \ll 2/T_D$$

which means that half the locking bandwidth and the Doppler frequency shift have to be much less than twice the inverted time delay. If (28) is valid, the frequency ω can be written

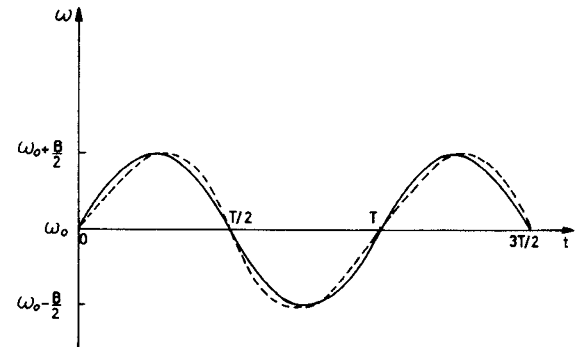

Fig. 6. Examples of frequency variations of a self-mixing Doppler radar. —— with one harmonic ω_D; ---- with two harmonics ω_D and $2\omega_D$.

$$\omega = \omega_0 + \frac{B}{2} \sin(\omega_D t + \varphi_0). \qquad (31)$$

The frequency of the oscillator thus oscillates with a deviation $B/2$ and a modulation frequency of ω_D. A plot is shown in Fig. 6. If (28) is not valid there will be distortion of the pure sinusoidal variation. An example is shown in Fig. 2.

It is interesting to note that the oscillator is not locked by the Doppler-shifted signal, even if the Doppler shift frequency is less than half of the locking bandwidth.

The previous discussion leads to the conclusion that for $B/2 \ll 2/T_D$ and $\omega_D \ll 2/T_D$ the higher harmonics of ω_D can be neglected. The moving reflector can therefore be represented by a small time-dependent load impedance:

$$\Delta Z_L = r(\cos \omega_D t + j \sin \omega_D t) \qquad (32)$$

which, together with the noise sources, is the generator in the system.

ACKNOWLEDGMENT

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